

Thermodynamics of quark matter with a chiral imbalance

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We show how a scheme of rewriting a divergent momentum integral can conciliate results obtained with the Nambu–Jona-Lasinio model and recent lattice results for the chiral transition in the presence of a chiral imbalance in quark matter. Purely vacuum contributions are separated from medium-dependent regularized momentum integrals in such a way that one is left with ultraviolet divergent momentum integrals that depend on vacuum quantities only. The scheme is applicable to other commonly used effective models to study quark matter with a chiral imbalance, it allows us to identify the source of their difficulties in reproducing the qualitative features of lattice results, and enhances their predictability and uses in other applications.

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I. INTRODUCTION

There has been an increased interest recently in the study of how a chiral imbalance of right-handed and left-handed quarks can influence the phase diagram of quantum chromodynamics (QCD). There are many good reasons for this interest. For instance, the nontrivial nature of the vacuum of non-Abelian gauge theories in general, and of QCD in particular, allows for the existence of topological solutions like instantons and sphalerons. While instantons describe the quantum tunneling between different vacua, sphalerons are classical solutions describing transitions going above the barrier between the vacua. Sphaleron processes are unsuppressed at high temperatures [1, 2] and, from the Adler-Bell-Jackiw anomaly, they can generate, in the context of QCD, an asymmetry between the number of left- and right-handed quarks. Such a chirality imbalance is expected to occur in event-by-event C - and CP -violating processes in heavy-ion collisions [3, 4]. Moreover, in off-central collisions a magnetic field is created and the presence of a chiral imbalance gives rise to an electric current along the magnetic field, whose effect is to produce a charge separation, an effect dubbed chiral magnetic effect (CME) in the literature— see, e.g., Refs. [5–8] for recent reviews and references therein. The CME effect is not restricted to QCD, it extends over a wide range of systems, e.g., hydrodynamics and condensed matter systems [9–15], and it has been actually observed in many recent condensed matter experiments [16], which makes it of much wider interest in physics.

The effects of a chiral imbalance in the phase diagram of QCD can be studied in the grand canonical ensemble by introducing a chiral chemical potential μ_5 through a term $\mu_5 \bar{\psi} \gamma_0 \gamma_5 \psi$ in the QCD Lagrangian density [4]. Besides of the intrinsic motivation in the context of the physics of heavy-ion collisions, there have been interesting suggestions [17, 18] that the phase diagram of QCD in the $T - \mu_5$ plane could be in principle mapped into the real phase diagram in the $T - \mu$ plane, where μ is the usual

quark baryon chemical potential, a feature that would help to pinpoint the expected critical end point (CEP) of QCD — see Refs. [19, 20] for opposite views. More important, however, is the fact that QCD in the presence of a chiral chemical potential is free from the sign problem and, therefore, amenable to Monte Carlo sampling in lattice simulations, contrary to the case of QCD in the presence of a baryon chemical potential, which has the sign problem. Hence, there is hope that lattice simulations of QCD with μ_5 can be used as a possible benchmark platform for comparing different effective models used in the literature. In this respect, it is intriguing that models that have been very successful in describing many features predicted by universality arguments and lattice simulations for the chiral transition in QCD at nonzero T and μ , have difficulties in reproducing, even at a qualitative level, recent lattice results [21, 22] for the chiral critical transition temperature T_c at finite μ_5 . For instance, predictions based on Nambu–Jona-Lasinio (NJL)-type of models [17, 23–27] and quark linear sigma models [17, 28] find that T_c decreases with μ_5 , while the lattice results of Refs. [21, 22] find T_c increasing with μ_5 .

A nonzero quark condensate mixes right- and left-handed quarks and has the effect of decreasing the chiral asymmetry. Therefore, as one forces a system to increase the right-left asymmetry by increasing μ_5 , one expects that the quark condensate will increase and, therefore, T_c is expected to increase likewise. This is because addition of left- and right-handed quarks to a system, in amounts controlled by μ_5 , favors quark-antiquark pairing, that is, increases the quark condensate [29]. Universality arguments in the large N_c limit (where N_c is the number of color degrees of freedom) [30] also predict a T_c increasing with μ_5 . Some recent studies using phenomenological quark-gluon interactions in the framework of the Dyson-Schwinger equations for the quark propagator [19, 20] and nonlocal finite-range NJL models [31, 32] find a T_c increasing with μ_5 . Both types of models have in common the feature of having a momentum-dependent quark mass function, in contrast to a constant mass in contact-

interaction models. A qualitative agreement with the lattice results for T_c was also found in Ref. [33], by using a nonstandard renormalization scheme in the quark linear sigma model.

Given the prominent role played by NJL type of models in providing insight into the problem of the chiral phase transition, it is important to identify the sources of their failure in reproducing the qualitative features of lattice simulations for the μ_5 dependence of T_c . In the present work we pursue such a study. Our analysis is based on a proper separation of medium effects from divergent integrals, so that all divergent integrals are the same as those that appear in vacuum, i.e., at $T = 0$ and $\mu_5 = 0$. This is motivated by a similar situation in studies of color superconductivity with NJL models, in that the traditional treatment based on cutoff regularization leads to a decreasing superconducting gap for high μ , while the separation of vacuum effects from μ -dependent divergent integrals leads to results in agreement with model-independent predictions [34]. We show that a similar effect is at play here, since μ_5 appears explicitly in divergent integrals. As such, a decreasing T_c with μ_5 seems to be a result of improper separation of medium effects from the vacuum contributions, thus subject to a dependence on how these divergent terms are regularized. This is also similar to the case of magnetized quark matter, where unphysical spurious effects are eliminated by properly disentangling the magnetic field contributions from divergent integrals [35, 36].

Our regularization procedure in expressing all divergent integrals in terms of integrals that appear in the vacuum is very simple and, once the divergent vacuum integrals are fixed to reproduce physical quantities in vacuum, our results predict an increasing T_c with μ_5 . This result is a simple consequence of the ability of writing all divergent integrals in terms of integrals that appear in the vacuum. Although we use a NJL model — see, e.g. Refs. [37, 38] for reviews and references — as an explicit example, the procedure applies equally well for other effective models for QCD, like the Polyakov–Nambu–Jona-Lasinio (PNJL) model [39] that includes the Polyakov loop contribution.

The remainder of this paper is organized as follows. In Sec. II we describe the regularization scheme that makes the vacuum ultraviolet momentum terms independent of the medium effects and its implementation in the context of the NJL model at finite chiral chemical potential and temperature. In Sec. III we contrast the results obtained in the context of this medium separation scheme with the traditional cutoff one. Our conclusions and final remarks are presented in Sec. IV.

II. THE NJL MODEL WITH A CHIRAL IMBALANCE

The NJL model, with a chiral chemical potential included, has the Lagrangian density given by

$$\mathcal{L} = \bar{\psi} (i\not{\partial} - m_c + \mu_5 \gamma^0 \gamma^5) \psi + G \left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2 \right], \quad (2.1)$$

where G is the coupling, m_c is the current quark mass ($m_c = 0$ in the quiral limit) and ψ represents a flavor isodoublet, N_c -plet quark field — a sum over flavors, $N_f = 2$, and color degrees of freedom, $N_c = 3$, is implicit. The mean-field thermodynamic potential $\Omega(M, T, \mu_5)$ for the model is a function of the dynamical quark mass $M \equiv M(T, \mu_5)$, given by the gap equation $M = m_c - 2G \langle \bar{\psi}\psi \rangle$, as

$$\Omega(M, T, \mu_5) = \Omega_0(M, \mu_5) - 2N_f N_c T \sum_{s=\pm 1} \int \frac{d^3 k}{(2\pi)^3} \ln \left[1 + e^{-\omega_s(k)/T} \right], \quad (2.2)$$

where Ω_0 has no explicit T dependence,

$$\Omega_0(M, \mu_5) = \frac{(M - m_c)^2}{4G} - N_f N_c \sum_{s=\pm 1} \int \frac{d^3 k}{(2\pi)^3} \omega_s(k), \quad (2.3)$$

and $\omega_s(k) = \sqrt{(|\mathbf{k}| + s\mu_5)^2 + M^2}$ are the eigenstates of the Dirac operator with helicity $s = \pm 1$. Note that while the second term on the right-hand side of Eq. (2.2) is ultraviolet (UV) finite, the momentum integral in Ω_0 is UV divergent and requires a regularization prescription. Ω_0 depends explicitly on μ_5 and implicitly on T , through its dependence on M . To analyze the gap equation, one will need an integral that is the derivative with respect to M^2 of the momentum integral in Eq. (2.3); it can be expressed in the form

$$\frac{\partial}{\partial M^2} \left[\int \frac{d^3 k}{(2\pi)^3} \omega_s(k) \right] = \int_{-\infty}^{+\infty} \frac{dk_4}{2\pi} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{k_4^2 + \omega_s^2(k)}, \quad (2.4)$$

where we have introduced the four-momentum component k_4 (in Euclidean space) for convenience. In order to make explicit the vacuum contribution to the integral, we use three times in sequence the identity [40]

$$\frac{1}{k_4^2 + \omega_s^2(k)} = \frac{1}{k_4^2 + k^2 + M_0^2} + \frac{k^2 + M_0^2 - \omega_s^2(k)}{(k_4^2 + k^2 + M_0^2) [k_4^2 + \omega_s^2(k)]}, \quad (2.5)$$

such that the integrand in Eq. (2.4) can be rewritten in the form [34]

$$\frac{1}{k_4^2 + \omega_s^2(k)} = \frac{1}{k_4^2 + k^2 + M_0^2} - \frac{A_s(k)}{(k_4^2 + k^2 + M_0^2)^2} + \frac{A_s^2(k)}{(k_4^2 + k^2 + M_0^2)^3} - \frac{A_s^3(k)}{(k_4^2 + k^2 + M_0^2)^3 [k_4^2 + \omega_s^2(k)]}, \quad (2.6)$$

where we have defined $A_s(k) = \mu_5^2 + 2sk\mu_5 + M^2 - M_0^2$ and M_0 is the quark mass in the vacuum (i.e., computed at $T = 0, \mu_5 = 0$). Equation (2.6) can be verified by direct algebraic manipulation. Note that, when substituting it back of Eq. (2.4), the first term on the right-hand side in Eq. (2.6) leads to a quadratically divergent integral, the two next terms are proportional to a logarithmically divergent integral, and the last term leads to a finite integral. It is important to note that the divergent integrals are the same as those arising in the vacuum, as there is no explicit or implicit dependence on T or μ_5 in their integrands. Thus, one can regularize the integrals as we wish, as, e.g., by a three-dimensional momentum cutoff Λ , and fix Λ by fitting a vacuum physical quantity. The last term, being finite, can be integrated without any momentum cutoff, the same way as we do for the second term of Eq. (2.2), the explicitly temperature dependent term.

It is at this point where our approach differs from all previous calculations: In the traditional approach, the left-hand side of the identity in Eq. (2.6) is used in Eq. (2.4) and a momentum cutoff is used to perform the integral with an integrand that depends explicitly and implicitly on medium quantities, μ_5 and $M \equiv M(T, \mu_5)$, while when using the right-hand side of the identity, Eq. (2.6), one obtains divergent integrals that are independent from the medium, i.e., they are dependent on the vacuum quark mass M_0 only. In other words, by using the identity in Eq. (2.6), medium and vacuum dependences can be explicitly disentangled from the integrands of the divergent integrals and, therefore, do not get cutoff by any regulator. In the rest of this work we refer to this regularization procedure as “medium separation scheme” (MSS), while the usual treatment of the divergent integrals as “traditional regularization scheme” (TRS).

Earlier works that have applied TRS in different effective models of QCD [17, 18, 23–26, 28] have found a critical temperature T_c for chiral symmetry restoration that decreases with μ_5 . They also find a CEP on the phase diagram (μ_5, T_c) . Recent lattice results [21, 22] obtained instead a T_c increasing with μ_5 and a transition that is only a crossover. The idea behind the MSS method is not new [40], as already mentioned, it was used previously in a similar situation that occurs with the NJL in the study of color superconductivity [34], and it actually resembles [41] the Bogoliubov, Parasiuk, Hepp, Zimmermann renormalization scheme [42], in that the integrand of a divergent amplitude is manipulated to isolate the divergence without applying an explicit regulator.

The dynamical quark mass M is determined self-consistently by solving the gap equation derived from Eq. (2.2) which, with the help of Eq. (2.6), becomes

$$\begin{aligned} \frac{M - m_c}{4N_f N_c G M} = & I_{\text{quad}}(\Lambda, M_0) \\ & + (2\mu_5^2 - M^2 + M_0^2) I_{\text{log}}(\Lambda, M_0) \\ & - \frac{2\mu_5^2 + M^2 - M_0^2}{8\pi^2} + \frac{M^2 - 2\mu_5^2}{8\pi^2} \ln\left(\frac{M^2}{M_0^2}\right) \\ & - \sum_{s=\pm 1} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{\omega_s(k)} \frac{1}{e^{\omega_s(k)/T} + 1}, \end{aligned} \quad (2.7)$$

where $I_{\text{quad}}(\Lambda, M_0)$ and $I_{\text{log}}(\Lambda, M_0)$ denote the quadratically and logarithmically UV divergent integrals, respectively,

$$I_{\text{quad}}(\Lambda, M_0) = \int^\Lambda \frac{d^4 k}{(2\pi)^4} \frac{1}{k_4^2 + k^2 + M_0^2}, \quad (2.8)$$

and

$$I_{\text{log}}(\Lambda, M_0) = -\frac{\partial}{\partial M_0^2} I_{\text{quad}}(\Lambda, M_0), \quad (2.9)$$

where Λ denotes the regularization parameter used in the divergent integrals. Note that the quark mass dependence of both $I_{\text{quad}}(\Lambda, M_0)$ and $I_{\text{log}}(\Lambda, M_0)$ is through the vacuum quark mass M_0 . We reiterate that once a regularization scheme is chosen, I_{quad} and I_{log} are fixed by fitting vacuum properties; for example, I_{quad} and I_{log} can be expressed in terms of the quark condensate $\langle \bar{q}q \rangle$, the leptonic decay constant f_π and the pion mass m_π . Once G , m_c and the dynamical quark mass in the vacuum M_0 are chosen to fit those physical quantities, the integrals are fixed.

III. NUMERICAL RESULTS

We fix the parameters of the model by using as input $f_\pi = 92.3$ MeV, $m_\pi = 0.140$ GeV and $\langle \bar{q}q \rangle^{\frac{1}{3}} = -0.250$ GeV, and use a three-dimensional cutoff to evaluate the vacuum divergent integrals. A good fit is obtained with $m_c = 5.37$ MeV, $G = 4.75$ GeV⁻² and $\Lambda = 0.660$ GeV. The constituent quark mass is found to be $M_0 = 0.302$ GeV.

In Fig. 1 we show the results for the dynamical quark mass M as a function of μ_5 in the case where $T = 0$. The results at a fixed temperature (below the critical temperature for chiral symmetry restoration) are qualitatively similar. We show the results for both the TRS and MSS regularizations explained above. Here a note of caution is in order regarding values of μ_5 close to Λ . One should keep in mind that the NJL model, being a non-renormalizable model, has an intrinsic energy scale and its predictions of phenomena driven by dynamics occurring at energies higher than that scale should be taken with great caution. Although the precise limit of validity can be a matter of discussion, as it might depend on type

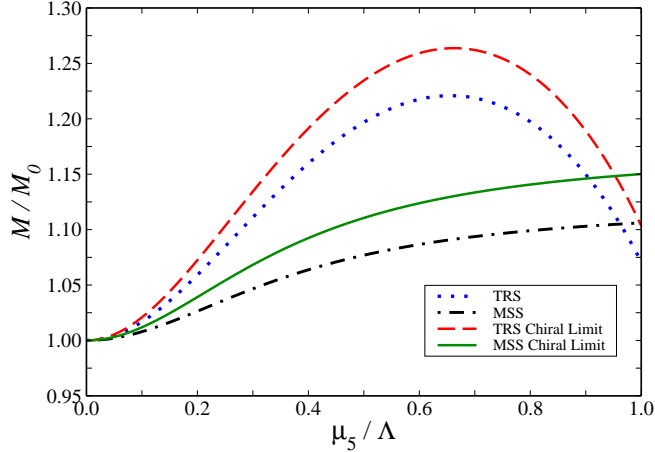


Figure 1. The zero temperature quark mass M , normalized by its respective vacuum value M_0 , as a function of chiral chemical potential μ_5 . The results obtained using the TRS regularization (see text) are given by the dashed (chiral limit) and dotted lines. In the MSS regularization the results are given by the solid (chiral limit, $m_c = 0$) and dash-dotted lines.

of observable or physical process at study, the value for that scale is commonly assumed in the literature to be the cutoff Λ . In view of this and in order to avoid misinterpretations, we have restricted the value of μ_5 in Fig. 1 to be at most Λ . Note that even though the TRS scheme seems to indicate that the chiral chemical potential initially strengthens dynamical chiral symmetry breaking (DCSB), the behavior changes at around $\mu_5 \simeq 0.6\Lambda$, beyond which it starts to disfavor DCSB. However, in the MSS scheme, DCSB is always strengthened by the chiral chemical potential; with all the required proviso just mentioned, we remark that this continues to be true for values of μ_5 larger than Λ . Thus, we see that in the TRS regularization, the tendency of the chiral chemical potential is to weaken the chiral symmetry breaking beyond $\mu_5 \gtrsim 0.6\Lambda$, while in the MSS regularization the tendency is always to strengthen it. This change of behavior, which is directly related on how the vacuum dependent term on μ_5 is handled, of course reflects on how the critical temperature changes too. This is explicitly shown in Fig. 2.

The values of T_0 , for the critical (T_c) and pseudo-critical (T_{pc}) temperatures for chiral symmetry restoration evaluated at $\mu_5 = 0$ used in Fig. 2, are given in Tab. I.

Table I. Values of critical (T_c) and pseudo-critical (T_{pc}) temperatures for the chiral symmetry restoration at $\mu_5 = 0$

	T_c (GeV)	T_{pc} (GeV)
TRS	0.165	0.177
MSS	0.169	0.183

In Fig. 2 we show the results for the critical tempera-

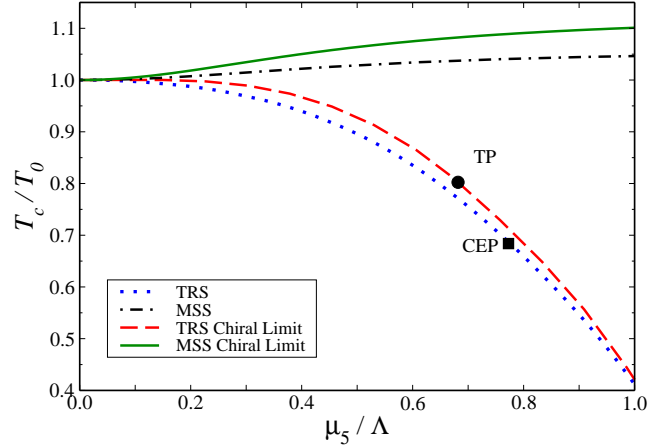


Figure 2. The critical temperature T_c , normalized by $T_0 = T_c(\mu_5 = 0)$, as a function of μ_5 . The black dot indicates a tricritical point in the chiral limit, while the square indicates the critical end point, both in the TRS regularization case (see text for a detailed explanation).

ture T_c as a function of μ_5 for the two forms of treating the divergent integrals. In the TRS regularization, we find a critical end point (CEP) that separates a crossover line from a first-order transition. In the chiral limit ($m_c = 0$) it is instead a tricritical point (TP), which separates a line of second-order phase transition from one of first-order. However, in the MSS regularization both the TP and the CEP are absent. The transition is a crossover (note that in this case T_c in Fig. 2 indicates, technically, the pseudo-critical temperature), while in the chiral limit the transition is second order throughout. In conformity with the behavior seen for the dynamical quark mass in Fig. 1, because of the deleterious effect of the chiral chemical potential on the breaking of chiral symmetry, T_c decreases in the TRS regularization. But in the MSS regularization one sees that T_c always increases with μ_5 . This is in qualitative accordance with the recent results from the lattice [21, 22] and also with more sophisticated nonperturbative treatments, e.g., like the ones used in Refs. [19, 20]. As far the absence of the TP (in the chiral limit) or the CEP in the MSS regularization is concerned, this is also seen in the results obtained from the earlier lattice results [43] and also with the more recent ones, where no CEP (or TP) has been found. One should, however, mention here that the lattice results in Refs. [21, 22] should be taken with some caution, as they were obtained for a very large pion mass, $m_\pi = 418$ MeV, while here we used the physical value of $m_\pi = 140$ MeV. It is known that some quantities (for example the behavior of the quark condensate as a function of an external magnetic field) may depend heavily on the pion mass. So we cannot rule out the possibility that the nonexistence of a CEP in those lattice results could be an artifact of the large pion masses used in those numerical studies.

The increase of the pseudo-critical and the critical (in the chiral limit) temperatures are again consistent with the behavior seen for the dynamical quark mass in the MSS regularization shown in Fig. 1.

Finally, as already remarked, being the NJL model an effective model, it has an intrinsic scale that limits its validity. A natural choice for this scale can be taken for example as being the regularization or cutoff scale in the present case, Λ , and we do expect that the results should be reliable for values of μ_5 not too above this scale. We note from the results of both Figs. 1 and 2 that the differences between the TRS and MSS regularization schemes are already significant for values of $\mu_5 \ll \Lambda$. In particular, the differences between the (pseudo-) critical temperature T_c in the TRS and MSS schemes are already apparent for values of μ_5 as low as around $\mu_5 \approx 0.3\Lambda$, where the tendency of growth for T_c is already clear.

IV. CONCLUSIONS

Our results show that a way of conciliating results for the chiral critical transition line obtained with NJL models and recent lattice results, when in the presence of a chiral imbalance, might be closely connected on how the UV momentum integrals are treated in these models. These same results also show that one can eliminate this discrepancy by a proper separation of medium effects from the integrand of the divergent integrals that require regularization. All resulting divergent integrals are the same as those that appear in the vacuum, i.e., at $T = 0$ and $\mu_5 = 0$. By this proper separation of medium effects from the divergent vacuum integrals, we have obtained results for the critical temperature dependence with the chiral chemical potential that are in qualitative agreement with physical expectations, in that μ_5 is a catalyst of DCSB [29] and, therefore, an increasing critical temperature as a function of μ_5 should be expected. Moreover, our results are in line with the arguments of Ref. [29] that the ultraviolet cutoff Λ , used with a TRS, effectively cuts important degrees of freedom near the Fermi surface leading to an incorrect result for the critical temperature as a function of μ_5 . We also have qualitative agreement with lattice results regarding the absence of a CEP. Note, however, as we have already remarked, the comparison should be taken with caution, given the large pion mass used in those lattice studies. Likewise, the position and even (non)existence and of a CEP can depend heavily on the pion mass. Nevertheless, we must also point out that recent studies [19, 20] based on a renormalizable, nonperturbative scheme based on the Dyson-Schwinger equations of QCD also do not find a critical end point in the phase diagram (T, μ_5) —see also discussions in Ref. [27]. While definite lattice results with

physical pions masses are still missing, it is fair to say that there is strong evidence that there is no CEP in the phase diagram of quark matter with a chiral imbalance. In the MSS regularization, we found that the transition is a crossover in the physical case of $m_0 \neq 0$, while in the chiral limit, $m_0 = 0$, it is second-order throughout.

One additional bonus of properly separating medium effects from divergent vacuum momentum integrals, is the fact that once the parameters of the model are chosen to fit physical quantities in vacuum, the divergent integrals are fixed and they are not changed when studying T and μ_5 effects. This is simply a consequence of making the UV divergent momentum integrals, $I_{\text{quad}}(\Lambda, M_0)$ and $I_{\text{log}}(\Lambda, M_0)$, to depend only on vacuum quantities. Thus, in the present case where we have chosen a three-dimensional momentum cutoff Λ for the UV divergent integrals, both I_{quad} and I_{log} are fixed once the values of Λ and M_0 are fitted to the physical quantities. Even though arguments can be made against such a separation of vacuum and medium effects in the NJL model, we believe that in some cases, such a strategy, in the present case given by the MSS regularization scheme, seems to be important for capturing the right physics with the model. Though we have offered arguments in favor of the MSS procedure, it is clear that more work is welcome, in particular, more work on different regulators is needed.

We believe that this same methodology that we have employed in this work will also be relevant in any other problem where this mixing of medium and regularization might be present. Our results, thus, indicate a way of improving the predictability of these effective models, which are so useful in our effort to explain one of the most difficult problems in physics today, i.e., the structure of the QCD phase diagram.

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